Mathematics Standard
Articulated By Grade Level

INTRODUCTION

Mathematics is a highly interconnected discipline. The need to understand and use a variety of mathematical strategies in multiple contextual situations has never been greater. Utilization of mathematics continues to increase in all aspects of everyday life, as a part of cultural heritage, the workplace, and in the scientific and technical communities. Today's changing world will offer enhanced opportunities and options for those who thoroughly understand mathematics.

The Mathematics Standard Articulated By Grade Level describes a connected body of mathematical understandings and competencies that provide a foundation for all students. They define the understanding, conceptual knowledge, and skills that students are to acquire.

Communication, problem solving, reasoning and proof, connections and representation are the process standards as described in the *Principles and Standards for School Mathematics* from the National Council of Teachers of Mathematics (NCTM). These process standards are interwoven within all the content strands of the Arizona Mathematics Standard. The process standards emphasize ways to acquire and use the content knowledge.

Mathematics education should enable students to fulfill personal ambitions and career goals in an informational age. In the NCTM *Principles and Standards* document it asks us to "*Imagine a classroom, a school, or a school district where all students have access to high-quality, engaging mathematics instruction. There are ambitious expectations for all, with accommodations for those who need it*".[1] The Arizona Mathematics Standard Articulated by Grade Level is intended to facilitate this vision.

BACKGROUND

The state Board of Education adopted the Arizona Academic Standards in 1996 to define what Arizona's students need to know and be able to do by the end of twelfth grade. Developed by committees comprised of educators, parents, students, and business and community leaders, these standards were written in grade-level clusters with benchmarks at grades 3, 5, 8, and high school.

RATIONALE

Requirements in the *No Child Left Behind Act of 2001* (NCLB) and the need to do a periodic review of the state academic standards prompted the decision by the Arizona Department of Education to refine and articulate the academic standards for mathematics and reading by grade level. This refinement and articulation project was started in July 2002, and was completed in March 2003.

[1] National Council of Teachers of Mathematics, Principles and Standards for School Mathematics, NCTM Publications, Reston, VA, 2000, p. 3.

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METHODOLOGY

Work teams for mathematics were formed of a representative sample from around the state to include large and small schools, rural and urban schools, and ethnic diversity. Included were national mathematics consultants, university professors, community members, and test company consultants who advised the teams. The goal was to articulate, or align, the current academic standards by grade level. The mathematics standard was articulated K-10 to align with the state requirement of two years of high school mathematics.

The mathematics articulation team utilized the National Council of Teachers of Mathematics *Principles and Standards* as a reference in the development of the revised mathematics standards. Additionally, the ACHIEVE, *Foundations for Success, Mathematics Expectations for the Middle Grades* document, served as a guide to evaluate the level of achievement expectations for Arizona's students.

The articulation teams created draft documents with the existing standards and performance objectives articulated, or aligned, to the appropriate grade level. Over a period of months, these teams and smaller sub-committees of the teams refined the documents. Reasonableness, usefulness, and appropriateness were the guidelines for the articulation process. The measurability of each performance objective was considered.

External reviews by nationally recognized consultants brought a broader perspective to the articulation process. Internal reviews by university and local experts provided additional validation.

Another important step in the project was the gathering of public comment. In December 2002, drafts of the Standards Articulated By Grade Level, along with a survey to gather feedback, were posted on the Arizona Department of Education website. This provided the public with easy access to the documents, and a survey allowed reviewers a means for submitting comments. The public and all educators had the opportunity to submit comments and suggestions, either electronically or in writing, until the survey closing date of January 31, 2003. Additionally, six public hearings were held in January throughout the state offering further opportunities for public input.

After all the public comments were collected and organized by topic, the teams met one last time to determine what modifications to the standards documents would be appropriate. Upon completion of the refinements, glossaries, rationales, and crosswalks were developed to assist educators with the transition from the 1996 Arizona Academic Standards to the Mathematics Academic Standard Articulated By Grade Level.

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ORGANIZATION OF THE MATHEMATICS STANDARD

The Mathematics Standard Articulated By Grade Level are divided into five main strands:

- Number Sense and Operations
- Data Analysis, Probability, and Discrete Mathematics
- Patterns, Algebra and Functions
- Geometry and Measurement
- Structure and Logic.

Each strand is divided into concepts that broadly define the skills and knowledge that students are expected to know and be able to do. Under each concept are performance objectives (PO) that more specifically delineate the tasks to be taught and learned.

The document is designed so that teachers can read the performance objectives across grade levels to incorporate learning from previous, current and future grade levels. Viewing the mathematics standard document from left to right helps the teacher to see the mathematics continuum across the grade levels. Looking down each individual column enables a teacher to see the performance objectives that students are expected to know and be able to do at any grade level.

This organization does not imply that teaching and learning of mathematics should be fragmented or compartmentalized. Mathematics is a highly interconnected discipline; topics from all five mathematics strands need to be continuously integrated as needed to make meaning and connections to other concepts and performance objectives.

The order of the strands, concepts, and performance objectives (POs) in the mathematics standard document are not intended to be a checklist for mathematics instruction. Mathematical concepts develop with a spiraling of skills that are interconnected and dependent on each other and this is reflected in the standard. Effective instruction often incorporates several performance objectives into an integrated experience of learning for the student.

Due to the nature of the content, some performance objectives are repeated in subsequent grade levels. However, the intent is that the complexity, depth, and difficulty of the performance objective content must increase from one grade level to the next.

Strand One: Number Sense and Operations

Number Sense is the understanding of numbers and how they relate to each other and how they are used in specific context or real-world application. It includes an awareness of the different ways in which numbers are used, such as, counting, measuring, labeling, and locating. It includes an awareness of the different types of numbers, such as, whole numbers, integers, fractions, and decimals and the relationships between them, and when each is most useful. Number sense includes an understanding of the size of numbers, so that students should be able to recognize that the volume of their room is closer to 1,000 than 10,000 cubic feet.

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Students develop a sense of what numbers are: to use numbers and number relationships, to acquire basic facts, to solve a wide variety of real-world problems, and to estimate to determine the reasonableness of results.

Concept 1: Number Sense

Understand and apply numbers, ways of representing numbers, the relationships among numbers and different number systems.

Concept 2: Numerical Operations

Understand and apply numerical operations and their relationship to one another.

Concept 3: Estimation

Use estimation strategies reasonably and fluently.

Strand 2: Data Analysis, Probability, and Discrete Math

This strand requires students to use data collection, data analysis, statistics, probability, systematic listing and counting, and the study of graphs. This prepares the student for the study of discrete functions, fractals and chaos, and to make valid inferences, decisions, and arguments.

Discrete mathematics is a branch of mathematics that is widely used in business and industry. Combinatorics is the mathematics of systematic counting. Vertex-edge graphs are used to model and solve problems involving paths, networks, and relationships among a finite number of objects.

Concept 1: Data Analysis (Statistics)

Understand and apply data collection, organization, and representation to analyze and sort data. This is considered to be the analysis and interpretation of numerical data in terms of samples and populations.

Concept 2: Probability

Understand and apply the basic concepts of probability. This is the field of mathematics that deals with the likelihood that an event will occur expressed as the ratio of the number of favorable outcomes in the set of outcomes divided by the total number of possible outcomes.

Concept 3: Discrete Mathematics (Systematic Listing & Counting)

Understand and demonstrate the systematic listing and counting of possible outcomes. This field of mathematics is generally referred to as Combinatorics.

Concept 4: Discrete Mathematics (Vertex-Edge Graphs)

Understand and apply the concepts vertex-edge graphs and networks. This field ties in graph theory with practical problems.

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Strand 3: Patterns, Algebra, and Functions

Patterns occur everywhere in nature. Algebraic methods are used to explore, model and describe patterns, relationships, and functions involving numbers, shapes, iteration, recursion, and graphs within a variety of real-world problem solving situations. Iteration and recursion are used to model sequential, step-by-step change.

Algebra emphasizes relationships among quantities, including functions, ways of representing mathematical relationships, and the analysis of change.

Concept 1: Patterns

Identify patterns and apply pattern recognition to reason mathematically. Students begin with simple repetitive patterns of many iterations. This is the beginning of recursive thinking. Later, students can study sequences that can best be defined and computed using recursion

Concept 2: Functions & Relationships

Describe and model functions and their relationships. For example, distribution and communication networks, laws of physics, population models, and statistical results can all be represented in the symbolic language of algebra.

Concept 3: Algebraic Representations

Represent and analyze mathematical situations and structures using algebraic representations. Algebraic representation is about abstract structures and about using the principles of those structures in solving problems expressed with symbols.

Concept 4: Analysis of Change

Analyze change in a variable over time and in various contexts such as, qualitative change, quantitative change, and the idea that slope represents the constant rate of change in linear functions, and functions that have non-constant rates of change.

Strand 4: Geometry and Measurement

Geometry is a natural place for the development of students' reasoning, higher thinking, and justification skills, culminating in work with proofs. Geometric modeling and spatial reasoning offer ways to interpret and describe physical environments and can be important tools in problem solving. Students use geometric methods, properties and relationships, transformations, and coordinate geometry as a means to recognize, draw, describe, connect, analyze, and measure shapes and representations in the physical world.

Measurement is the assignment of a numerical value to an attribute of an object, such as the length of a pencil. At more-

sophisticated levels, measurement involves assigning a number to a characteristic of a situation, as is done by the consumer price index. Understanding what a measurable attribute is and becoming familiar with the units and processes that are used in measuring attributes, is a major emphasis in this strand.

Concept 1: Geometric Properties

Analyze the attributes and properties of two- and three-dimensional shapes and develop mathematical arguments about their relationships (in conjunction with strand 5, concept 2).

Concept 2: Transformation of Shapes

Apply spatial reasoning to create transformations and use symmetry to analyze mathematical situations.

Concept 3: Coordinate Geometry

Specify and describe spatial relationships using coordinate geometry and other representational systems.

Concept 4: Measurement - Units of Measure
- Geometric Objects

Understand and apply appropriate units of measure, measurement techniques, and formulas to determine measurements.

Strand 5: Structure and Logic

This strand is unique to the Arizona Mathematics Standard and might be considered an extension of problem solving. Students draw from the content of the other four strands to devise algorithms and analyze algorithmic thinking. Strand One and Strand Three provide the conceptual and computational basis for these algorithms. Logical reasoning and proof draws its substance from the study of geometry, patterns, and analysis to connect remaining strands. Students use algorithms, algorithmic thinking, and logical reasoning, both inductive and deductive, as they make conjectures and test the validity of arguments and proofs. They evaluate situations, select problem solving strategies, draw logical conclusions, develop and describe solutions and recognize their applications.

Concept 1: Algorithms and Algorithmic Thinking

Use reasoning to solve mathematical problems in contextual situations. Determine step-by-step series of instructions to explain mathematical processes.

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Concept 2: Logic, Reasoning, Arguments, and Mathematical Proof

Evaluate situations, select problem solving strategies, draw logical conclusions, develop and describe solutions, and recognize and describe their applications. Develop mathematical arguments based on induction and deduction, and distinguish between valid and invalid arguments.